Logarithms (Log Means "FIND THE EXPONENT")

Logs cause headaches. I have to admit it. Students find logs hard. Why? I think in part it's because of the word "log". It seems to have no connection with what it represents in math. Then again, students seem to find exponents easy. So when you see "log", think, even read, "**FIND THE EXPONENT!**"

For example, $\log_2 16$:

Find the exponent you need with base 2 to get a value of 16. Hmm...2 times 2 times 2 times 2...**FOUR**! You see, that's not so hard! It's not like beating your head against a log. Sorry. Here are the rules.

$\log_a(xy) = \log_a x + \log_a y$	$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$	$\log_a\left(x^y\right) = y\log_a x$	
$\log_a 1 = 0$	$\log_a a = 1$	$\log_a a^{-1} = -1$	$\log_a\left(a^x\right) = x$

DON'T confuse $\log_a\left(x^y\right)$ with $(\log_a x)^y$. For example,

$$\log_2\left(4^3\right) = \log_2\left(\left(2^2\right)^3\right) = \log_2\left(2^6\right) = 6 \text{ while } \left(\log_2 4\right)^3 = \log_2\left(2^2\right)\cdot\log_2\left(2^2\right)\cdot\log_2\left(2^2\right) = 2^3 = 8.$$

Change of base formula: $\log_a x = \dfrac{\log_b x}{\log_b a}$ and in particular, $\log_a b = \dfrac{1}{\log_b a}$

Special bases: $\log x$ **MEANS** $\log_{10} x$ $\quad$ $\ln x$ **MEANS** $\log_e x$

Example 1) Evaluate: (a) $\log 1000$ (b) $\log_2 \frac{1}{16}$ (c) $\log_3\left(27^4\right)$ (d) $\left(\log_3 27\right)^4$

Solution (a) $\log 1000 = 3$ (b) $\log_2 \frac{1}{16} = -4$ (c) $\log_3\left(27^4\right) = 4(3) = 12$ (d) $\left(\log_3 27\right)^4 = 3^4 = 81$

Example 2) Expand using properties of logs: (a) $\log_3\left(\frac{x^3 y^4}{z^5}\right)$ (b) $\log\left((x^2+y^2)(x^2-y^2)\right)$

Solution (a) $\log_3\left(\frac{x^3 y^4}{z^5}\right) = 3\log_3 x + 4\log_3 y - 5\log_3 z$

(b) $\log\left((x^2+y^2)(x^2-y^2)\right) = \log(x^2+y^2) + \log(x^2-y^2) \overset{\boxed{\text{optional}}}{=} \log(x^2+y^2) + \log(x-y) + \log(x+y)$

Example 3) Change $\log_9 x$ to log base 4, log base 10, and log base *e*.

Solution $\log_9 x = \dfrac{\log_4 x}{\log_4 9} = \dfrac{\log x}{\log 9} = \dfrac{\ln x}{\ln 9}$

Two for you.

1)(a) Expand using properties of logs: $\log\left(a^{-2}b\sqrt{c}\right)^3$ (Hint: rewrite $\sqrt{c}$ as $c^{1/2}$.)

(b) Combine using properties of logs: $3\ln x - 4\ln y + 1$ (Hint: use $1 = \ln e$.)

2) Change $\log e$ (that is $\log_{10} e$) to base e.

Answers 1)(a) $-6\log a + 3\log b + \dfrac{3}{2}\log c$ (b) $\ln\left(\dfrac{ex^3}{y^4}\right)$ 2) $\log e = \dfrac{1}{\ln 10}$